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THE DETERMINATION OF TRAFFIC IN A ROAD
NETWORK - AN ECONOMIC APPROACH

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#### 1. Introduction

one of the striking features of traffic is its resemblance to physical flows. The relations between speed and volume and such phenomena as the propagation of waves of stopping and acceleration are understandable from the mechanics of motion. However appealing this physical view of the matter, it neglects the purposive aspect of traffic, which becomes particularly important when attention is shifted from the single road to an entire network; that is to say, if we are interested in understanding the generation and distribution of traffic within a system of interconnected roads.

The point of view to be adopted in this context is that vehicles are operated by people who 1) have a range of choices available to them and 2) are motivated by economic considerations in their decisions. This element of choice prevails, even though, once committed to the road, the movement of vehicles is governed largely by traffic conditions around them. For there is always the question, why people anticipating these constraints, and particularly the irritations of congestion, still choose to go when, where and at the speed they go.

The problem of accounting for the element of individual choice in traffic behavior is not of merely theoretical interest; for on its solu-

2/ Cf. Herrey, E. M. J. and Herrey, H., "Principles of Physics Applied to Traffic Movements and Road Conditions," American Journal of Physics, vol. 13, p. 1, (February, 1945).

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tion it will depend whether more exact and scientifically satisfying methods for traffic prediction can be developed. Three levels of the problem may be distinguished: 1) the structure of individual choices affecting traffic, 2) the repercussions on choices from traffic conditions, and 3) the problem of traffic equilibrium in a network.

Traffic engineering research has made important contributions to these through empirical and theoretical analysis of specific problems. What is lacking perhaps is recognition of the economic aspects of the problems involved and a general framework in terms of which the various results may be integrated. The present paper suggests such a framework on the basis of some recent developments in mathematical economics. It gives a verbal outline of a model of traffic behavior and is part of a broader study on resource allocation problems in transportation, the mathematical treatment of which will be presented elsewhere.

In standard economic fashion the investigation will be carried out on two levels: 1) that of the individual road user whose decisions are made under the assumption of given traffic conditions that are considered unaffected by his own particular choice; and 2) that of all traffic participants taken together, in which the interdependence of the various individual decisions is explicitly brought out. It will be noted that this partition ignores the possibility of road users operating on a scale large enough to affect traffic conditions noticeably, as may be the case for large trucking enterprises with respect to traffic in the vicinity of their terminals, or for such organizations as the armed forces. Rather this analysis envisages the ordinary user of the road, and it is believed that the cases mentioned before are perhaps best conceived as deviations from the norm, to be treated by alteration of the basic model.

#### 2. Cost Factors

The decisions of road users that affect the pattern and extent of traffic flows are mainly: whether at all to travel to a given point by road, which route to choose, which free speed, and which risks to take in passing slower vehicles. The variables that influence their decisions in turn are: the money cost of travel, chiefly for fuel and oil; the time spent; the nuisance of congestion; and the risks envisaged. In the perspective of the economist, these various items constitute a bundle of discommodities, that is, of inconvenience undergone in order to obtain a certain end; transportation to a given point. If the driver picks a particular route and chooses to behave in a certain way as regards free speed and pagsing, this is interpreted as indicating a preference for one among alternative bundles of these discommodities.

In economic theory it is assumed that preferences satisfy some minimal requirements of consistency: it must be impossible to have a chain of commodity bundles, each of which is preferred to the previous one, with the last and the first bundle being the same. It can be shown that consistent preference implies a valuation of the commodities (or discommodities) in money terms, provided that one particular (dis)commodity is expressed in money. For the present decision problem, this implies a valuation of the whole trip, of time, risk, and congestion muisance in money terms, which is brought out if the road user is confronted with a sufficient set of alternatives. The fact that the individual himself may not be aware of these implicit valuations is no objection as long as his choices are consistent in the above sense.

In commercial transportation, the values of time and sometimes of risk are in fact given in explicit form. The per unit value of time, for instance, is composed of drivers' wages and the time cost to cargo, the

detailed composition of which need not interest us here. The point is that while the values of time, etc., may be different for commodity and passenger transportation, in fact different for any two individuals, it is nevertheless true that a definite (money) value is assigned to time by every road user. This justifies considering the decisions of road users, once the choice to go by road is given as the outcome of cost minimization in terms of these concealed but definite money values. A few words for clarification may be added on the items of risk and congestion nuisance.

The notion of risk pertains to the probabilities of accidents of the various kinds. Its valuation, therefore, ultimately imputes values to health and human life. Valuation of life in money terms is implicit in all economic relationships that involve danger to human life. Since the strict enforcement of the "safety first" principle would mean in effect a standstill of most economic activity, a compromise is made which puts certain high, but finite, values on human life and health; and this is the case too for modern highway transportation. Theoretically, the cost of risk could be obtained by discounting the (negative) money values of death and various injuries, and the direct money costs involved, with the probabilities of their occurrence. For all practical purposes a measure of risk is afforded by the probability of an accident of any kind. An average loss - value is then assigned to an accident by each individual; and the situation is now quite parallel to that of time cost valuation.

<sup>3/</sup> Our objective of ascertaining the existence of valuations implicit in drivers' choices of time, etc., is clearly different from, but, of course, not unrelated to, the problem of assigning some plausible value to time for purposes of road construction policy.

The problem of defining congestion nuisance in an operational way can be treated only on the second level of the analysis. Here we remark only that the reduction of average speed from the average speed at which vehicles move when alone on the road furnishes a simple and unambiguous measure. Again an individual component must enter into the valuation of congestion nuisance.

Beyond the formal logics of economic decisions, can economics explain the substantive aspects of prevalent choices? One would think here of an explanation of the value to individuals of transportation between principal traffic generating points. This is indeed one of the tasks that location theory purports to answer. But, while this branch of economics may often help our intuition with clarifying arguments, its present state is still one of inadequacy to cope with the more complex problems of interlocal flows of people and commodities.

In the few cases where an economic assessment of the value of transportation between given locations is possible, this happens because there exists a geographic differential in commodity prices or in earning possibilities capable of further analysis. In general, economics cannot explain why peoples values are what they are, since tastes must be assumed given, except possibly in psychology.

If explanation fails, how then about the possibilities of at least measuring values? One would be interested not in each individual's values, but of course in the valuations of the "typical" individual, or more precisely, the frequency distribution of valuations over the particular population of road users. Since no one is able to verbalize his valuations, measurement can take place only by inference from actions. But since all the pertinent values are operative simultaneously, a difficult problem of

identification arises. That is to say, the given observations are compatible with more than one set of values behind the actions of drivers. Fortunately, it is not always necessary for prediction purposes to go back to the individual values. Traffic behavior can be formalized at an intermediate level, the second level of the present analysis, in a way sufficient for most practical purposes. Yet it is essential for an understanding of the significance of these simplifications that the theoretical analysis be explicit up to the stage of individual decisions and valuations.

#### 3. Drivers' Choices

3.1 The decision whether to go some place at all or have commodities shipped to some location, depends on economic factors mostly outside the highway network, and not belonging to the domain proper of traffic engineering. All we need to say here is that the expected returns from transportation must not be less than the total costs, composed of all the factors ascertained before: money, time, risk, and congestion nuisance. Only as far as commodities are concerned an objective measure of returns is sometimes available in terms of the geographical price difference for the commodities. With respect to personal transportation, the element of individual valuation (of the returns) is involved again.

If alternative means of transportation are available, individual demand for transportation by road depends on the relative costs. Discussion of the (total) demand function for road transportation belongs to the second level of the analysis. Given that the individual decides to travel by road between certain points, the returns side is fixed and all his further decisions may be viewed as aimed at minimizing costs.

3.2 The elements that enter into the choice among routes are: money cost as reflected primarily in geographical distance and possible road tolls, time losses through delays on congested roads and at intersections,

the inconvenience of congestion, and finally risks. A crucial point for the analysis is that in many cases the geographically closest route and the optimal route with respect to the factors mentioned do not coincide. This is the real reason why economic analysis of traffic must be in terms of networks rather than with reference to a single road or intersection. Since we have a case of discrete choice among particular alternatives, comparison need by made only of the total cost figures. Thus the only data that enter into the driver's choice are the amounts and values of the elements previously mentioned, — time, money, risk, and congestion.

3.3 A distinction must be made for the problem of speed selection, depending on whether the number of passings to be made enters as a consideration or not. (Simplifying, we may call these the cases of the congested and of the uncongested road). The notion of desired or free speed refers to an idealized version of the second situation: where the vehicle is the only one on the road. It is with respect to the determination of the desired speed, that the apparatus of economic theory is most readily applicable. Roughly, the argument runs like this.

Even if no legal speed limits existed, the speed of traffic would of course not be indeterminate. If it is not to be assumed that driving speed is the invariable product of habits (in which case an explanation of its formation would still be required) a rational element will have to be admitted. Now from a certain speed on every increase of speed by (say) 10 mph. will lead to a more than proportionate increase in risk. Similarly from some speed on, money costs will increase more than proportionately.

For example, John Beakey and F. B. Crandall, The Effect of Surface Type, Alignment and Traffic Congestion of Vehicular Fuel Consumption, Oregon State Highway Commission, Highway Department, Technical Bulletin No. 5, 1937.

Furthermore, since time consumed per mile is the inverse of speed, the savings in time cost per unit increase of speed necessarily decrease with increasing speed. The three factors combine therefore to decrease the advantages (or increase the cost) of additional speed with increasing speed. At the point of optimum the total cost is at a minimum and its derivative with respect to speed therefore is zero. This is to say that with a small increase in speed the sum increase of time loss x value of time + increase of money cost - increase of risk x value of risk (negative) should be equal to zero.

Now in order to evaluate these derivatives, we have to know more accurately how time, money cost, and risk depend on speed. Time obviously is inversely proportional to (free) speed, provided speed remains constant on a given road, the proportionality factors being the length of the road. Fuel cost, the essential part of samey cost, as a function of speed, has been measured in the article referred to. Sith respect to risk we are not aware of any comparable measurements.

The equation stating that the net increment of cost from an increase in speed should be sere, permits to say something about the effect of structural changes on the free speed. Thus if through technological change, the rate of increase of money cost with speed is lowered (e.g. by means of an "overdrive"), or if in response to a rise of the standard of living the relative value of time of an individual is increased, we conclude that a premium is put on higher speeds. In the latter case, however, the disutility of risk increases also in proportion as the value of life is estimated at a higher level.

A legal speed limit enters either as an unquestioned constraint on the set of possible speeds and then tends to coincide with the selected speed for all those drivers whose desired speed would be higher, or otherwise it gives rise to an increase in money cost via the costs of fines discounted at a risk factor for being caught. It appears in the latter case, that as the speed limit is lowered, the amount of its violation that appears advantageous to the individual must increase, unless the controls or the fines are raised correspondingly.

3.4 One way of going from here to the case of a congested road is to introduce congestion nuisance as a function of speed into the analysis.

In a more explicit approach two features emerge: the choice of the free speed is influenced by the anticipation of the number of passings to be made; and secondly, average speed is decreased by the delay in passing slower vehicles. While the first effect may be negligible, some consideration is necessary for the second one, as to the conditions accepted for passing. Here we have a situation where only time and risk enter as relevant cost parameters. The observable variable is the so-called critical gap, or the minimal time distance to the nearest vehicle in opposite direction for which the individual is willing to pass. If risk as a function of the critical gap could be calculated, and if also the average waiting times for the various gaps were known, a driver's critical gap could serve to evaluate his risk preference in terms of his value on time. As of now no satisfactory solution to either problem has appeared.

#### 4. Capacity and Demand Functions

On the first level of the analysis we have asked how traffic conditions affect individual choices. On the second level the problem is somewhat the reverse: how do the various individual choices of route and speed combine to govern traffic conditions? This requires the study of three specific problem areas dealing respectively with speed, waiting

times, and demand for traffic as functions of traffic flows: namely, 4.1 the road capacity functions, 4.2 the intersection delays, and 4.3 the problems of diverted and generated traffic. A fourth pertinent problem, concerning the accident rate as a function of traffic flows, has been omitted here. 5

l.1 We shall call the relation between speed and flow on a particular road the capacity of that road. Nearly all students of highway capacity have started off their investigations by attempts to determine this relation, but they have for the most part used it only as a stepping stone to capacity itself which they usually define as a number representing the highest possible (or "comfortable" or "practical") flow under certain given conditions. Since this search for a "best" point on a speed-flow curve has led to a great deal of arbitrariness, we wish to retain the whole curve and call it the capacity, or capacity curve. The description in 5 below will illustrate why it is difficult to select the best point on the curve, how it could in fact be done, but even more importantly why the selection of such a point is premature if done solely on the basis of capacity considerations without reference to the demand for road use.

In the older literature on the subject attention was focussed on the case where all vehicles traveled at the same speed. As the speed went up, the spacing increased more than proportionately, and the resulting flow of vehicles past a certain point was found to increase to a maximum and then decrease as the gain in speed was more than offset by

This is not to deny the importance of the problem. But it was felt that the contribution that differences in risks owing to differences in flow on alternative routes would make to drivers choices of such routes, is of second order of magnitude in the equilibrium to be described below.

the increase in spacing. For quite obvious reasons this uniform-speed capacity curve was never found to be very useful. People do not all drive at the same speed ordinarily, and in those cases where they do some of them at least wish to go faster and will pass the cars ahead of them as soon as the opportunity arises. This leads to a bunching of traffic which the curve described does not take into account. Of course it should be said that in the case of a large scale closely directed traffic movement, such as a military convoy, these shortcomings may disappear.

The capacity notion to be used here is the very different one developed by O. K. Normann and his associates in the Euresu of Public Roads, and described in the Highest Capacity Manual. For a particular road a capacity curve is derived supplementally which relates everage speed and flow. The curves which have been published show that flow increased only at the expense of a darked reduction of average speed. Further theoretical work is mecassary before a really good explanation of this mechanism can be given, but it seems to be fairly elser that higher flows inhibit passing, and flow reaches a maximum when all cars are traveling at very close to the same speed; which then is necessarily a relatively slow one.

It is not very clear in Normann's work whether these curves must be determined empirically for each road of interest or whether a study of typical two-lane roads, typical three-lane roads, etc., would suffice. The point is an important one because aspects other than the physical characteristics of the road may affect the shape of the curve. The question is how important are these other elements, and how much do they differ between roads in different places. One of the things we have in mind is the distribution of the desired speed discussed in 3, or as it is cometimes called, the "free-speed distribution," which tells us what

fraction of the traffic wants to travel at 45 mph., what fraction at 60 mph., etc. It is easy to see that the scatter of this distribution has an effect, for suppose that all cars wish to travel at about 60 mph.; then there will be very little necessity for passing, and very little reduction in speed when passing is prevented. On the other hand, if desired speeds are widely dispersed, say half the traffic wants to move at 30 mph. and half at 60 mph., then passing will be frequent when it is possible, and delays will be severe when it is not. For the first distribution, average speed will go down less with increases in flow than for the latter distribution.

From a theoretical point of view it would seem that each road should have a capacity curve of its own. For not only do the distributions of desired free appeals depend on the physical road conditions, but also those of the distributions of gaps accepted for passing, and perhaps even the general preferences for speed of the populations of drivers that use the road. For suppose that there is a choice between a longer and faster and a slower but fore direct route. Then it is to be expected that the drivers with preference for greater speed tend to accumulate on the fast road.

Earlier it was mentioned that the muisance aspect of congestion may have some influence on a driver's choice of route. A little more can now be said on this point. Suppose the time consumed in traveling over alternative routes between the same origin and destination is the same. It may still be that drivers prefer one route because the traffic on the other makes it more bothersome to drive. Judging from one's own experience, this sounds like it might be an important element to consider, and some attention in fact has been given it in the literature on capacity. 6/

<sup>6/</sup> O. K. Normann, "Preliminary Results of Highway Capacity Studies," Public Roads, v. 19, p. 228 (Pebruary, 1939).

The only suggestion to be made here is that if in attempting to explain drivers' choices of routes it is found necessary to include some measure of this nuisance factor, the ratio of average actual speed to average desired speed might not be a bad one. It gives an indication of the extent to which drivers are prevented by the presence of traffic from traveling at the speeds they would like to go, and it is a means of distinguishing between routes which, from the viewpoint of time, distance, risk, and money cost, are identical.

4.2 The capacity of an intersection, like that of a road, will again be described in terms, not of a number representing an upper limit, but of a relation between flows through the intersection and the average delays incurred by the various flows, given a certain signalling system.

For what we are chiefly interested in, as before, is not the upper limit to flow or some "practical" limit, but rather the way in which delays depend on flows. In principle, flows and delays for different directions of traffic should be distinguished, for there are, of course, circumstances under which an increase in the flow in one direction causes little or no delay to itself, but greatly augments the delays suffered by vehicles in the crossflow. If we wish to predict traffic movements, it is essential to know how the delays are allocated.

Intensive empirical studies, like Greenshields, of the behavior of vehicles at intersections should be very useful in deriving such capacity curves in a case-by-case fashion. However, more theoretical work is necessary before much can be said of a general nature beyond such intuitively appealing but qualitative assertions as "delays should increase

<sup>7/</sup> Bruce D. Greenshields, Donald Schapiro, and Elroy L. Erickson, Traffic Performance at Urban Street Intersections, Yale Bureau of Highway Traffic, Technical Report No. 1 (1947).

more than proportionately with flows. Recent developments in the study of random processes (systems whose law of change is given in terms of probability distributions) lead one to believe that the prospects in these directions are fairly good.

tween given points is first of all a function of its price, which we have split up before into its various components: money costs, money values of time, risks and congestion. As with other demand functions, in economics, the demand for traffic can be estimated only by observations for different sets of prices. In this respect "before and after-studies" can be a source of valuable information. But pertinent detailed data are rare. Only in the short-run, that is, while we consider no changes in network capacities and the costs of travel are constant, is the demand for road transportation adequately described by origin-destination figures. As soon as changes of capacity are admitted, changes of cost result and we are confronted with an induced demand for traffic. Failure to adequately take this "generated traffic" into account has led to the embarrassingly frequent underestimation of traffic on new constructions.

Besides on the prices for road transportation, the demand for traffic depends on other parameters. It has been suggested that the amount of traffic between two cities is inversely proportional to their distance and directly proportional to the algebraic product of their population num-

<sup>8/</sup> See a forthcoming article on intersection delays by C. B. Winsten of the Cowles Commission.

<sup>9/</sup> John O. Stewart, The Development of Social Physics, American Journal of Physics, Vol. 18, No. 5, May, 1950, p. 248, and references.

bers, in the fashion of Newton's law of gravitation. But clearly this can be valid only as a first approximation, since it entirely disregards the particular economic (industrial) structure of the communities in question. Knowledge of the regional economy, if quantified in suitable form, should permit a more accurate estimate in individual cases.

#### 5. Traffic Equilibrium

A system of flows and speeds in a road network will be said to be in equilibrium if traffic conditions and drivers' choices are consistent. Suppose the flow on a given road happens to be 600 vehicles per hour at a certain moment. We know from the capacity curve that the average speed corresponding to this flow will be, say 30 mph. It will not be higher. for the density of vehicles will be too great to allow the frequency of passing that a higher speed calls for; it will not be lower either, for the free-speed distribution upon which the capacity curve is based ensures that when passing opportunities arise there are enough fast drivers present to bring the average speed back up to 30 mph. Now suppose that at this speed of 30 mph., 700 vehicles find this road a profitable one to travel. Traffic on the road will increase and the speed we started with will not be maintained. If only 500 vehicles per hour found the road to be a good route, then the speed we started with would increase. Neither of these situations represents an equilibrium; only if at 30 mph. just 600 vehicles picked this road to travel would we have equilibrium on this road. When the number of drivers choosing particular roads and speeds is such as to maintain the existing combinations of speeds and flows on all roads, that is, when demand and traffic conditions inviting demand correspond with each other throughout the network, then we have a system in equilibrium. In view of the periodicity of traffic over the day, week and year, equilibrium conditions can be realized only in the short run, or with regard to the overall pattern of a longer period.

An equilibrium is said to be stable if, any deviation from the flow and speed pattern induces a return to the original pattern, provided always that the deviation is not too large. It turns out that with the stated properties of road and intersection capacity curves and under the assumption that traffic demand decreases - or at least does not increase - with costs, there exists a stable traffic equilibrium provided drivers seek to minimize cost as defined in the previous section.

With this equilibrium approach in mind, the difficulties involved in selecting "practical" capacities, or "best" points on capacity curves, become apparent. We might define as "best" that point on the capacity curve for which every vehicle on the road was making as good time as on competing routes. In possession of this knowledge, no driver at least would be dissatisfied with the thought that he could have done better. But of course this point is precisely the point of equilibrium; it will always occur, and the attempt to find it without analyzing origin-destination flows is to beg the whole question of what flows and speeds will actually occur.

Another way of defining this "best" point might be to state arbitrarily that average actual speed should be within, say, 10 mph. of average desired speed. The major shortcoming of such a capacity measurement as a criterion is that unless these speed margins had some precise relation to each other (10 mph. for all roads would not do in general) they would provide no guide whatsoever to the economics of road improvement. Only the equilibrium analysis could provide the proper margins, so again we are back where we started.

While from a purely theoretical point of view it is satisfying to consider in full complexity the interplay of choices that leads up to a traffic equilibrium, practical considerations require drastic simplifications in order to get results. For this reason we shall outline a simplified equilibrium model on the assumption that drivers choose the route which minimizes travel time. In other words we put a heavy weight on the individual's value of time. A corollary of this assumption (open to test) is that all traffic will distribute itself in such a way that if a particular flow between a given origin and destination uses more than one route the travel times for these different routes will be equalized. With this least-travel-time assumption, traffic in a given network depends on 1) the originations and destinations, that is to say, the demand functions for traffic; 2) the road and intersection capacity curves.

#### 6. Prediction

With this simplified equilibrium model in mind, we now turn to the principal purpose of economic traffic analysis, which is prediction. Prediction problems may be broken down into long-run and short-run problems. If we are wondering whether or not a new bridge or a new road should be built we must consider the effects of such construction on the decisions to buy cars and travel by road rather than, say, rail, and we must also know how many people will be induced to travel on the new routes who didn't travel at all before. The effect of permanent changes in the network and the decisions affected are long-run; they determine what was referred to above as "generated" traffic, [See 4.3] or more precisely, the demand functions for road transportation.

<sup>10/</sup> For a very interesting examination of this hypothesis, see D. L. Trueblood, "The Effect of Travel Time and Distance on Freeway Usage," Public Roads, v. 26, p. 241 (February, 1952).

In the short-run we have such problems as the direction of traffic at special events, or the revision of signalling and channelization arrangements to handle a temporary detour of traffic from a road under repair. But also with respect to long-run problems, i.e. network changes, it is useful to solve the problem first only with regard to the short-run decisions of drivers and then to find out how the result is changed in the longrun. Thus in predicting the changes caused by a proposed new construction of some sort we would first wish to find how the change will alter current decisions as to routes and speeds. That is to say, what traffic will be diverted. On this we may superimpose then the effects of traffic generated. In the short-run the only choices left to drivers are those of route and speed, flows of traffic from each point of origin to each point of destination being fixed, if we classify the decision to travel by road or some other means as a long-run choice. This is not to say that for any particular problem we know the numbers involved, but at least methods exist for estimating these origin-destination flows, and quite an extensive and interesting literature on this subject has grown up. The next step is to attempt a description of the resulting short-run traffic equilibrium which we suppose to prevail.

Seldom will it be necessary to derive the total flow and speed system from the start; more often, the problem is to predict the traffic flows after a given change in the network design. One way to go about this is first to consider a situation in which all the existing roads and intersections traffic speeds are those prevailing before the change, and for the new part speeds are desired speeds. Now for these speeds find for each origin-destination pair the quickest routes and then, on paper, distribute the corresponding flows over these quick routes. On some parts of the system the flows so derived will be different from the old flows (if the new construction makes any sense at all) because for some component

flows corresponds (via the capacity curve) a new average speed. Using these new speeds the procedure of distributing the flows is now repeated and a revised set of average speeds results. Ideally this procedure would be continued until no change occurred, as would be the case when the equilibrium solution had been obtained. Since, however, we are interested only in an approximate solution, a few steps of this procedure should be sufficient, provided that the alteration in network layout was not too big.

In that case, the existence of a traffic equilibrium ensures that this computational process comes to an end.

The only modification required by a long-run analysis is to replace the fixed origin-destination data by demand functions for transportation depending on cost, that is, travel times. The computational process is not changed in principle, but more detail is added and data of another kind are needed, viz. the demand functions. It is clear that these functions are much more difficult to estimate and are subject to greater error than the usual origin-destination data. One way to get at these functions is by extrapolation through an origin-destination study before and (well) after a significant change in the network. The evaluation of a "before and after" study of this kind calls itself for an equilibrium model.

#### 7. Conclusion

The practical purpose of an equilibrium model is prediction. But the purpose of prediction must be seen with reference to the ultimate end of traffic evaluation: the determination of the economically justified needs for road construction. The problem, what the highway network ought to be in order that it be of maximal benefit to the public, is a long-run economic problem of a high order of complexity. To put it differently,